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Estimates of Time Series Components of Road Traffic Accidents and Effect of Incomplete Observations: Mixed Model Case

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ABSTRACT

This study examines Buys-Ballot estimates of time series components of road traffic accidents and effect of incomplete observations in descriptive time series analysis. The ultimate objective of this study is therefore, to estimate trend parameters and seasonal indices using Buys-Ballot table with incomplete observations. Specific objectives are 1) to estimate the trend parameters and seasonal indices of the monthly number of road traffic accidents over the period under investigation. 2) to compare the estimates of trend parameters and seasonal indices with and without incomplete observations. 3) to determine the appropriate model. The methods adopted in this study are Regression Imputation (RI) when trend-cycle component is linear, Row Mean Imputation (RMI) and Buys-Ballot table for time series decomposition. The model structure used is mixed.

Keywords: Descriptive Time Series, Missing Data, Trend Parameter, Seasonal Indices, Mixed Model, Buys-Ballot Table.

1 INTRODUCTION

In time series analysis, a problem usually encountered in collection of data is missing values. Missing data may be practically impossible to obtain because of cost or time constraints. In order to derive estimates of value, many available options are left for the researcher. One of the available options is to replace them with mean of the series. Brockwell and Davis [1] presented the option that missing observations at the beginning or the end of the time series are simply ignored while intermediate missing observations are considered serious flaws in the input time series data. Hence, it interpolates values using interpolation algorithms: Linear, polynomial, smoothing, spline and filtering. Ljung [2] obtained an expression for likelihood function of the parameters in an autoregressive moving average (AMA) model when there are missing observations within the time series data. Luceno [3] extended the method by Ljung [2] for estimating missing observations and evaluating the corresponding likelihood function in scalar time series to the vector cases.

Seasonal component is simply define as regular periodic movements in time series data associated with the time of the year. Such movements are due to recurring events which take place annually. Many time series, such as sales figures and temperature readings, displays a variation which is annual in period. Cyclical component on other hand, is regarded as long oscillations appears to appreciable magnitude only in long period of time. However, if short period of time are involved, the trend component is jointly combined into cyclical Chatfield [4] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular/

residual component (e_t). Therefore, the decomposition models are

Additive Model

$$X_t = M_t + S_t + e_t \qquad (1)$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \qquad (2)$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \qquad (3)$$

This work will discuss only the mixed model which is used when the original time series contains very small or zero values. This study is restricted to time series with linear trend and admits mixed model using registered number of reported road traffic accidents over the period ten (10) years. The study isto estimate the trend parameters and seasonal indices using Buys-Ballot table with incomplete observations which takes into consideration the mixed model structure and linear trending curves. In achieving this objective, empirical example was employed. The data was taken from monthly records of number of road traffic accidents from the Federal Road Safety Officer in Owerri, Imo State, Nigeria over the period of January, 2009 to December, 2018. One hundred and twenty (120) registered road traffic accidents were considered over the period under investigation in which three (3) registered accidents were missing/ not accounted for. The observed series was transformed and the trend parameters and seasonal indices estimated using Methods of Row Mean Imputation (RMI) and Regression Imputation (RI). The missing observations will be estimated using this decomposition method and the entire process of estimation will be repeated without missing data. The appropriate model was determined for time series decomposition.

2. MATERIALS AND METHODS

2.1 Row Mean Imputation (RMI)

This method of replacing missing observation was proposed by Iwueze *et al.* [5] In this method, the row imputation computes the missing observation as the mean of the remaining observations in the period/year containing the missing observation. Hence, the missing data is estimated by;

$$RMI = \hat{X}_{(i-1)s+j} = \frac{1}{s-1}\left[\sum_{i=}^{j-1} X_{(i-1)s+j} + \sum_{u=j-1}^{s} X_{(i-1)s+u}\right] \qquad (4)$$

2.2 Regression Imputation (RI)

This method is one of the best method of replacing missing values. Regression Imputation method estimates the missing value by the estimate of the trend at the point of the missing observation. Thus, if the remaining observations of the series are employed to determine estimates of parameters trend and seasonal effect, then the estimates of the missing data at $(i-1)s + j$ of the trend-cycle component of the Regression Imputation are;

Linear Trend

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s + j] \qquad (5)$$

Quadratic Trend

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s + j] + \hat{c}[(i-1)s + j]^2 \qquad (6)$$

Exponential Trend

$$\hat{X}_{ij} = \hat{b}\, e^{\hat{c}[(i-1)s+j]} \qquad (7)$$

The estimates of the missing data by regression imputation for the additive and multiplicative and mixed models are;

For Additive model

$$\hat{X}_{ij} = \hat{M}_{(i-1)s+j} + \hat{S}_{(i-1)s+j} \qquad (8)$$

ii) For multiplicative model

$$\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \times \hat{S}_{(i-1)s+j} \qquad (9)$$

For mixed model

$$\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \times \hat{S}_{(i-1)s+j} \qquad (10)$$

2.3 Buys-Ballot Procedure for Time Series Decomposition

This method is based on the row, column and overall means of time series data arranged in a Buys-Ballot table with m rows and s columns, m is the number of observations in each column and s is the number of columns. For more details of

Buys-Ballot procedure, see Iwueze and Nwogu [6, 7] Iwueze and Ohakwe, [8] Dozie, [9] Dozie, *et al*, [10] Dozie and Ijomah. [11] For a series with linear trend, Nwogu, *et al*, [12] and Dozie *et al*, [10] obtained the row, column and overall means and variances for mixed model when trend cycle component of time series is linear given in Table 1.

2.4Linear Trend Component and Seasonal Indices

Using the expression in Table 1, the estimates of Trend parameters and seasonal indices are obtained for the linear trending curve.

$$\beta = b \tag{11}$$

$$\equiv \lfloor \alpha + \beta_j \rfloor S_j \tag{12}$$

Where. $\alpha = a + b\left(\frac{n-s}{2}\right)$, $\beta = b$

$$\hat{a} = \alpha - \hat{b}\left(\frac{n-s}{2}\right) \tag{13}$$

$$\hat{b} = \beta \tag{14}$$

Table 1: Summary of Row, Column and Overall Means and Variances of Buys-Ballot for Mixed Model

Measures	Linear trend-cycle component: $M_t = a + bt$, $t = 1, 2, ..., n = ms$
	Mixed model
$\bar{X}_{i.}$	$[a - bs + bsi] + \frac{b}{s}\sum_{j=1}^{s} jS_j + \bar{e}_{i.}$
$\bar{X}_{.j}$	$\left[a + b\left(\frac{n-s}{2}\right) + bj\right] * S_j + \bar{e}_{.j}$
$\bar{X}_{..}$	$a + b\left(\frac{n-s}{2}\right) + bC_1 + \bar{e}_{..}$
$\hat{\sigma}^2_{i.}$	$\{[(a + bs(i-1)) + bC_1]^2 + \text{var}[[a + bs(i-1)]S_j + bjS_j]\} + \sigma^2_1$
$\hat{\sigma}^2_{.j.}$	$\frac{b^2 n(n+s)}{12} S^2_j + \sigma^2_1$
$\hat{\sigma}^2_x$	$\frac{n}{n-1}\left\{\begin{array}{l}\frac{b^2(n^2-s^2)}{12} + \left[a^2 + 2ab\left(\frac{n-s}{2}\right) + \frac{b^2(n-s)(2n-s)}{6}\right]\text{Var}(S_j) \\ + 2b\left[a + b\left(\frac{n-s}{2}\right)\right]\text{Cov}(S_j, jS_j) + b^2\,\text{Var}(jS_j)\end{array}\right\} + \sigma^2_1$

Source: Nwogu, *et al*, (2019) and Dozie, *et al*, (2020)

Where

$$M_{(i-1)s+j} = X_{ij} = \hat{a} + \hat{b}[(i-1)s + j] \times S_j + e_{ij} \tag{15}$$

Estimates of α and β are obtained from the regression of $\bar{X}_{.j}$ on j and estimates S_j is

$$\hat{S}_j = \frac{\bar{X}_{.j}}{\hat{a} + \hat{b}\left(\frac{n-s}{2}\right) + b_j} \tag{16}$$

Table 2: Estimates of Trend Parameters and seasonal indices

Parameter	Model
	Mixed model
a	$\alpha - \hat{b}\left(\frac{n-s}{2}\right)$
b	β
S_j	$\frac{\bar{X}_{.j}}{\hat{a} + \hat{b}\left(\frac{n-s}{2}\right) + b_j}$

2.5 Estimation of Missing Data in the Transformed and Actual Time Series.

To estimate the missing data in the transformed series, the estimated trend parameters and seasonal indices for mixed model is given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s + j] \times \hat{S}_j \tag{17}$$

The exponent of the estimated missing data in the transformed series gives the estimate of the missing data in the actual data.

$$Actual \quad X_{ij} = e^{\hat{a} + \hat{b}[(i-1)s + j] \times \hat{S}_j} \tag{18}$$

2.6 Choice of Appropriate Model

Nwogu, *et al*, [12] and Dozie, *et al*, [10] proposed a test for choice between mixed and multiplicative models that can be based on the seasonal variances of the Buys-Ballot table. Hence, the null hypothesis to be tested is

H$_0$: $\sigma_j^2 = \sigma_{0j}^2$

and the appropriate model is mixed, against the alternative

H$_1$: $\sigma_j^2 \neq \sigma_{0j}^2$

and the appropriate model is not mixed, where

$\sigma_j^2 = (j = 1, 2, ..., s)$ is the actual variance of the jth column.

$$\sigma_{0j}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (19)$$

and σ_1^2 is the error variance, assumed equal to 1.

Under the null hypothesis, the statistic

$$\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{0j}^2} \qquad (20)$$

follows the chi-square distribution with $m-1$ degrees of freedom, m is the number of observations in each column and s is the number of columns, the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ contains the statistic (20) with 100 (1- α)% degree of confidence.

2.7 Bartlett's test for constant variance

To test the null hypothesis that the variances are equal, that is

$H_0 : \sigma_i^2 = \sigma_j^2$

against the alternative

$H_1 : \sigma_i^2 \neq \sigma_j^2$ for $i \neq j$

and at least one variance is different from others

Bartlett [13] has shown that, the statistic

$$T = \frac{(N-k)\ln S_p^2 - \sum (N_i - 1)\ln S_i^2}{1 + \frac{1}{3(k-1)}\left[\sum_{i=1}^{k} \frac{1}{(N_i - 1)} - \frac{1}{N-k}\right]} \qquad (21)$$

follows Chi-square distribution with (k – 1) degrees of freedom

Using the parameters of the Buys-Ballot table, $N = ms,\ k = s,\ N_i = m$, the statistic in (21) is then given as

$$T_c = \frac{(ms-s)\ln \hat{\sigma}_p^2 - \sum (m-1)\ln \hat{\sigma}_j^2}{1 + \frac{1}{3(s-1)}\left[\sum_{j=1}^{s} \frac{1}{m-1} - \frac{1}{ms-s}\right]}$$

$$= \frac{(m-1)\left[s \ln \hat{\sigma}_p^2 - \sum \ln \hat{\sigma}_j^2\right]}{1 + \frac{(s+1)}{3s(m-1)}} \qquad (22)$$

where ms is the total number of observations, m is the number of observations in each column and s is length of the periodic interval

3. Real life Example:

This section presents real life example based on monthly time series data on road traffic accident for the period of ten (10) years. One hundred and seventeen (117) registered road traffic accidents were considered from January to December 2009 to 2018 in which three (3) registered accidents were not accounted for. Estimates of trend parameters and seasonal indices with incomplete data are discussed in section 1. Section 2 contains the estimates of trend parameters and seasonal indices with complete data while choice of appropriate model is given in section 3. The time plots of actual and transformed series with missing data are given in figures3.1 and 3.2

Case 1: Estimate of trend parameters and seasonal indices with missing data

The linear trend of the row averages in the Buys-Ballot table with missing data is given as;

$$\bar{X}._j = 3.196 + 0.00036j \quad (23)$$

Using (13) and (14), $\hat{b} = 0.0004$

$$\hat{a} = 3.196 - 0.0004\left(\frac{120-12}{2}\right)$$

$$= 3.1744$$

Using (16), $\hat{S}_j = \dfrac{\bar{X}._j}{3.196 + 0.004_j}$

Case 2: Estimate of trend parameters and seasonal indices without missing data

Also, the linear trend of the row averages in the Buys-Ballot table without missing data is given as;

$$\bar{X}._j = 3.210 - 0.0013j$$

Table 3: Estimate of trend and seasonal indices with missing data

j	$\bar{X}_{.j}$	$\hat{S}_j$
1	4.442	1.3897
2	2.790	0.8727
3	2.770	0.8664
4	3.139	0.9818
5	3.058	0.9562
6	3.118	0.9749
7	2.894	0.9047
8	2.929	0.9155
9	3.092	0.9664
10	2.814	0.8794
11	2.998	0.9369
12	4.330	1.3554

Using (13) and (14)

$\hat{b} = 0.0013$, $\hat{a} = 3.2802$

Using (16), $\hat{S}_j = \frac{\bar{X}_{.j}}{3.210 + 0.013_j}$

Table 4: Estimate of trend and seasonal indices without missing data

j	$\bar{X}_{.j}$	$\hat{S}_j$
1	4.442	1.3843
2	2.790	0.8699
3	2.770	0.8640
4	3.210	1.0016
5	3.058	0.9546
6	3.118	0.9737
7	2.894	0.9041
8	2.929	0.9154
9	3.066	0.9586
10	2.814	0.8802
11	2.998	0.9381
12	4.330	1.3555

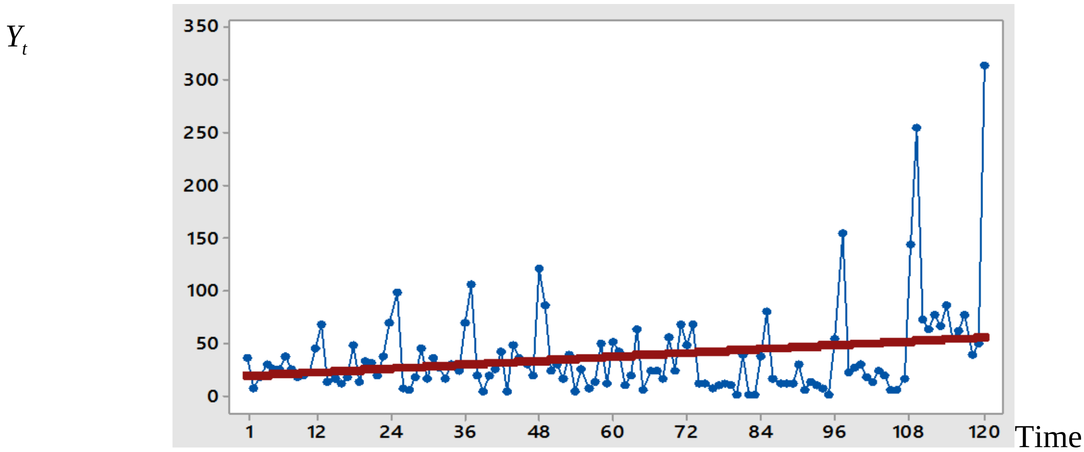

Figure 3.1: Time plot of the actual series on the number of road accidents between (2009-2018)

Table 5: Difference between trend parameters with and without missing data

Parameter	With missing data	Without missing data	Difference
$\hat{a}$	3.1744	3.2802	0.1058
$\hat{b}$	0.0004	-0.0013	0.0017

Where,

r_i = Number of observation in the rth row

c_j = Number of observation in the jth column.

Table 6: Periodic means and variances with and without missing data

Periods	With missing data			Withoutmissing data		
	r_i	$\bar{X}_i$	$\hat{\sigma}_i^2$	r_i	$\bar{X}_i$	$\hat{\sigma}_i^2$
1	10	3.181	0.2070	10	3.18	0.21
2	10	3.283	0.3940	10	3.28	0.39
3	10	3.205	0.6650	10	3.21	0.67
4	10	3.257	1.1190	10	3.26	1.12
5	9	3.1303	0.7600	10	3.19	0.79
6	10	3.284	0.5740	10	3.28	0.57
7	9	2.1887	1.3910	10	2.24	1.24
8	9	2.8011	0.9510	10	2.82	0.58
9	10	3.158	1.0140	10	3.16	1.01
10	10	4.400	0.3840	10	4.40	0.38
Overall Total		3.18882	0.7459		3.20	0.75

Table 7: Seasonal means and variances with and without missing data

Periods	With missing data			Without missing data		
	c_j	$\bar{X}_{.j}$	$\hat{\sigma}_{.j}^2$	c_j	$\bar{X}_{.j}$	$\hat{\sigma}_{.j}^2$
1	12	4.44	0.33	12	4.44	0.33
2	12	2.79	0.43	12	2.79	0.43
3	12	2.77	0.64	12	2.77	0.64
4	11	3.14	0.54	12	3.21	0.58
5	12	3.06	0.55	12	3.06	0.55
6	12	3.12	0.73	12	3.12	0.73
7	12	2.89	0.71	12	2.89	0.71
8	12	2.93	0.98	12	2.93	0.98
9	11	3.09	0.65	12	3.07	0.62
10	12	2.81	0.99	12	2.81	0.99
11	11	3.00	1.47	12	3.00	0.93
12	12	4.33	0.43	12	4.33	0.43
Overall Total		3.20	0.70		3.20	0.66

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total\ number\ of\ observation$$

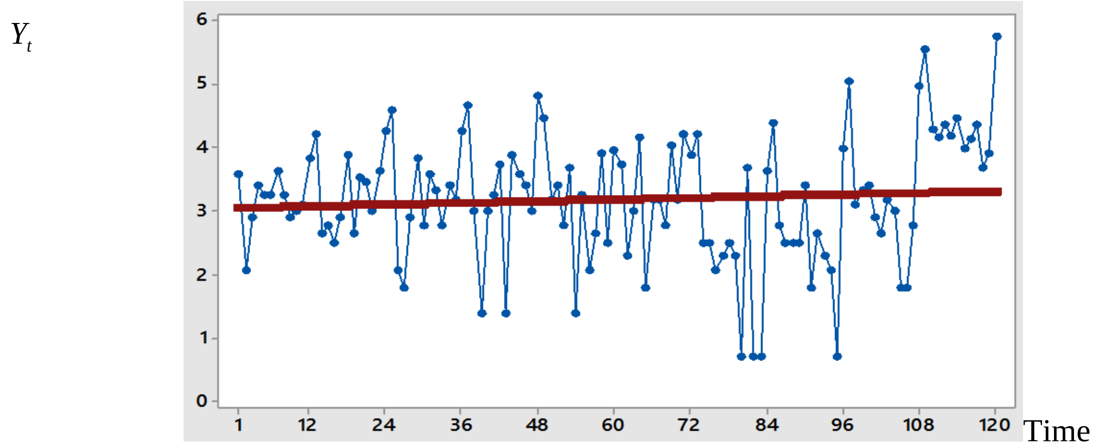

Figure 3.2: Time plot of the transformed series on the number of road accidents between (2009-2018)

Estimation of missing data in transformed series

Hence, the estimation of missing data in transformed and actual missing data

Using (14)

$$Actual\ \hat{X}_{5,4} = 3.1744 + 0.0004[(5-1)12+4]0.9818 = 3.1948$$

$$Actual\ \hat{X}_{7,9} = 3.1744 + 0.0004[(7-1)12+9]0.9664 = 3.2057$$

$$Actual\ \hat{X}_{8,11} = 3.1744 + 0.0004[(8-1)12+11]0.9369 = 3.2100$$

Using (15)

$$Actual\ \ value\ e^{3.1948} = 24.4052 \approx 24$$

$$Actual\ \ value\ e^{3.2057} = 24.6728 \approx 25$$

$$Actual\ \ value\ e^{3.2100} = 24.7791 \approx 25$$

Table 8: Difference between trend parameters with and without missing data

Parameter	With missing data	Without missing data	Difference
$\hat{a}$	3.1744	3.2802	0.1058
$\hat{b}$	0.0004	-0.0013	0.0017

Table 9: Difference between seasonal indices with and without missing data

Parameter	with missing data	without missing data	Difference
$\hat{S}_1$	1.3897	1.3843	0.0054
$\hat{S}_2$	0.8727	0.8699	0.0028
$\hat{S}_3$	0.8664	0.8640	0.0024
$\hat{S}_4$	0.9818	1.0016	-0.0198
$\hat{S}_5$	0.9562	0.9546	0.0016
$\hat{S}_6$	0.9749	0.9737	0.0012
$\hat{S}_7$	0.9047	0.9041	0.0006
$\hat{S}_8$	0.9155	0.9154	0.0001
$\hat{S}_9$	0.9664	0.9586	0.0078
$\hat{S}_{10}$	0.8794	0.8802	-0.0008
$\hat{S}_{11}$	0.9369	0.9381	-0.0012
$\hat{S}_{12}$	1.3554	1.3555	0.0001

Case 3: Choice of Appropriate Model

Here, the column variances of the Buys-Ballot table are obtained from appendix C and given in table 10. The modified Bartlett's test statistic given in (22) is used to determine whether the time series data admits additive model. The null hypothesis is rejected if T_c is greater than the tabulated value, which for α = 0.05 level of significance and m-1 degree of freedom equal to 19.7 or do not reject the null hypothesis otherwise.

Table 10: Seasonal effect (S_j) estimates of the column variances $\left(\hat{\sigma}_j^2\right)$ and calculated chi-square $\left(\chi^2_{cal}\right)$.

j	S_j	$\hat{\sigma}_j^2$	$\ln\hat{\sigma}_j^2$	χ^2_{cal}
1	2.63677	4085.50	8.3152	41.5962
2	0.54713	358.27	5.8813	82.6420
3	0.55732	298.89	5.7001	66.5087
4	0.8459	565.43	6.3376	55.3859
5	0.7101	352.18	5.8641	48.7301
6	0.7996	567.51	6.3413	62.1370
7	0.6244	251.38	5.5269	44.7924
8	0.6821	336.40	5.8183	50.3945
9	0.7476	525.79	6.2649	65.7497
10	0.6071	232.22	5.4477	43.7187
11	0.7174	381.51	5.9441	51.7429
12	2.5247	7080.40	8.8651	78.6214

From Appendix C and table 10

$$m = 10,\ \ s = 12,\ \ \hat{\sigma}_p^2 = 125296,\ \ \ln\hat{\sigma}_p^2 = 7.1333\ \ and$$

$$\sum_{j=1}^{s} \ln\hat{\sigma}_j^2 = 76.3066$$

Hence,

$$T_c = \frac{(9)[12\times 7.1333 - 76.3066]}{1 + \dfrac{13}{36(9)}} = 80.4083$$

When compared with the tabulated value (19.7), T_c is greater which indicatesthat the data does not admit the additive model.

Having confirmed that the appropriate model is not additive, we now choose between multiplicative and mixed model. According to the test proposed by Nwogu*et al*. [12] and Dozie, *et al*, [10] the null hypothesis that the data admits the mixed model is rejected if the test statistic given in (20) lies outside the interval $\left[\chi^2_{\frac{\alpha}{2}(m-1)},\ \chi^2_{1-\frac{\alpha}{2}(m-1)}\right]$ for which α=0.05 level of significance and m-1 = 9 degree of freedom, equals (2.7, 19.0) or do not reject null hypothesis otherwise.

From Appendix C and table 10, $\sigma_1^2 = 1,\ b = \ ,\ n = 120,\ m = 10,\ s = 12$

Hence, from (19)

$$\sigma^2_{.j} = (0.3102)^2 \times 120\left(\frac{120\times 12}{12}\right)S_j^2 + 1$$

And the calculated values of χ^2_{cal} given in table 10 were obtained. When compared with the critical value (2.7 and 19.0), the interval indicates that the data does not admit mixed model.

However, the result of the evaluation of data indicates that the data needs logarithmic transformation to meet the times series assumption in the distribution when column (monthly) variances of the logarithmic transformed data given in Table 11 are subjected to test for constant variance, the calculated Bartlett's test statistic (5.59) is less than the tabulated (19.7) at $\alpha = 0.05$level of significance and *m-1*= 9 degrees of freedom. This shows that the transformed data admits additive model.

Table 11:Column Variance ($\hat{\sigma}_j^2$) of number of road accident in a Buys-Ballot table

S/n	1	2	3	4	5	6	7	8	9	10	11	12
$\hat{\sigma}_j^2$	0.33	0.43	0.64	0.58	0.55	0.73	0.71	0.98	0.62	0.99	0.93	0.43
$\ln\hat{\sigma}_j^2$	-1.11	-0.85	-0.45	-0.55	-0.61	-0.31	-034	-0.02	-0.48	-0.01	-0.07	-0.84

From Appendix D and table 11

$$m = 10,\ s = 12,\ \hat{\sigma}_p^2 = 0.6596,\ \ln\hat{\sigma}_p^2 = 0.4161\ n = 120\ \ and\ \ \sum_{j=1}^{s}\ln\hat{\sigma}_j^2 = -5.6393$$

Thus,

$$T_c = \frac{(9)[12\times(-0.4161) - (-5.6396)]}{1 + \frac{13}{36(9)}} = 5.5855$$

4 Concluding Remark.

This paper has discussed the Buys-Ballot estimates of time series components of road traffic accidents and effect of incomplete data when trend cycle component is linear. The study provided the estimated missing data in descriptive time series analysis with linear trend and seasonality when more than one observations are missing in the Buys-Ballot table for seasonal time series. Also, the study show that the difference between trend parameters with and without missing data have insignificant effect but significant in seasonal indices of the Buys-Ballot table.

The appropriate time series model that best describe the pattern in the transformed data is additive. This shows that the original data series is multiplicative model.

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Appendix A: Buys-Ballot table for the actual data on number of road traffic accidents with missing observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	36	8	18	30	26	26	38	26	18	20	22	46	26.2	105.8
2010	68	14	16	12	18	48	14	34	32	20	38	70	32.0	425.5
2011	98	8	6	18	46	16	36	28	16	30	24	70	33.0	731.3
2012	106	20	4	20	26	42	4	48	36	30	20	122	39.8	1386.5
2013	86	24	30	-	40	4	26	8	14	50	12	52	30.2	555.2
2014	42	10	20	64	6	24	24	16	56	24	68	48	33.5	452.5
2015	68	12	12	8	10	12	10	2	-	2	2	38	18.0	408.0
2016	80	16	12	12	12	30	6	14	10	8	-	54	21.3	531.2
2017	154	22	28	30	18	14	24	20	6	6	16	144	40.2	2644.0
2018	254	72	64	78	66	86	54	62	78	40	50	314	101.5	7595.0
$\bar{X}_{.j}$	99.2	20.6	21.0	28.8	26.8	30.2	23.6	25.8	30.6	23.0	25.4	95.8		
$\sigma^2_{.j}$	4085.5	358.3	298.9	557.5	352.2	567.5	251.4	336.4	520.9	232.2	442.7	7080.4		

Appendix B: Buys-Ballot table for the transformed data on number of road traffic accidents with missing observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	3.58	2.08	2.89	3.40	3.26	3.26	3.64	3.26	2.89	3.00	3.09	3.83	3.18	0.21
2010	4.22	2.64	2.77	2.48	2.89	3.87	2.64	3.53	3.47	3.00	3.64	4.25	3.28	0.39
2011	4.59	2.08	1.79	2.90	3.83	2.77	3.58	3.33	2.77	3.40	3.00	4.25	3.21	0.66
2012	4.68	3.00	1.39	3.00	3.26	3.74	1.39	3.87	3.58	3.40	3.00	4.80	3.25	1.12
2013	4.45	3.18	3.40	-	3.69	1.37	3.26	2.08	2.64	3.91	2.48	3.95	3.10	0.76
2014	3.74	2.30	3.00	4.16	1.79	3.18	3.18	2.77	4.03	3.18	4.22	3.87	3.28	0.57
2015	4.22	2.48	2.48	2.08	2.30	2.48	2.30	0.69	-	0.69	0.69	3.64	2.31	1.39
2016	4.38	2.77	2.48	2.48	2.48	3.40	1.79	2.64	2.30	2.08	-	3.99	2.63	0.95
2017	5.04	3.09	3.33	3.40	2.89	2.64	3.18	3.00	1.79	1.79	2.77	4.97	3.16	1.01
2018	5.54	4.28	4.16	4.36	4.19	4.45	3.99	4.12	4.36	3.69	3.91	5.75	4.40	0.38
$\bar{X}_{.j}$	4.44	2.79	2.77	3.10	3.06	3.12	2.89	2.93	3.15	2.81	2.77	4.33	3.28	0.39
$\sigma^2_{.j}$	0.33	0.43	0.64	0.54	0.55	0.73	0.71	0.98	0.65	0.99	1.47	0.43	3.20	0.67

Appendix C: Buys-Ballot table for the actual data on number of road traffic accidents with missing observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	36	8	18	30	26	26	38	26	18	20	22	46	26.2	105.8
2010	68	14	16	12	18	48	14	34	32	20	38	70	32.0	425.5
2011	98	8	6	18	46	16	36	28	16	30	24	70	33.0	731.3
2012	106	20	4	20	26	42	4	48	36	30	20	122	39.8	1386.5
2013	86	24	30	16	40	4	26	8	14	50	12	52	30.2	555.2
2014	42	10	20	64	6	24	24	16	56	24	68	48	33.5	452.5
2015	68	12	12	8	10	12	10	2	40	2	2	38	18.0	408.0
2016	80	16	12	12	12	30	6	14	10	8	2	54	21.3	531.2
2017	154	22	28	30	18	14	24	20	6	6	16	144	40.2	2644.0
2018	254	72	64	78	66	86	54	62	78	40	50	314	101.5	7595.0
$\bar{X}_{.j}$	99.2	20.6	21.0	28.8	26.8	30.2	23.6	25.8	30.6	23.0	25.4	95.8		
$\sigma^2_{.j}$	4085.5	358.3	298.9	557.5	352.2	567.5	251.4	336.4	520.9	232.2	442.7	7080.4		

Appendix D: Buys-Ballot table for the transformed data on number of road traffic accidents with missing observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	3.58	2.08	2.89	3.40	3.26	3.26	3.64	3.26	2.89	3.00	3.09	3.83	3.18	0.21
2010	4.22	2.64	2.77	2.48	2.89	3.87	2.64	3.53	3.47	3.00	3.64	4.25	3.28	0.39
2011	4.59	2.08	1.79	2.90	3.83	2.77	3.58	3.33	2.77	3.40	3.00	4.25	3.21	0.66
2012	4.68	3.00	1.39	3.00	3.26	3.74	1.39	3.87	3.58	3.40	3.00	4.80	3.25	1.12
2013	4.45	3.18	3.40	2.77	3.69	1.37	3.26	2.08	2.64	3.91	2.48	3.95	3.10	0.76
2014	3.74	2.30	3.00	4.16	1.79	3.18	3.18	2.77	4.03	3.18	4.22	3.87	3.28	0.57
2015	4.22	2.48	2.48	2.08	2.30	2.48	2.30	0.69	3.69	0.69	0.69	3.64	2.31	1.39
2016	4.38	2.77	2.48	2.48	2.48	3.40	1.79	2.64	2.30	2.08	0.69	3.99	2.63	0.95
2017	5.04	3.09	3.33	3.40	2.89	2.64	3.18	3.00	1.79	1.79	2.77	4.97	3.16	1.01
2018	5.54	4.28	4.16	4.36	4.19	4.45	3.99	4.12	4.36	3.69	3.91	5.75	4.40	0.38
$\bar{X}_{.j}$	4.44	2.79	2.77	3.10	3.06	3.12	2.89	2.93	3.15	2.81	2.77	4.33	3.28	0.39
$\sigma^2_{.j}$	0.33	0.43	0.64	0.54	0.55	0.73	0.71	0.98	0.65	0.99	1.47	0.43	3.20	0.67

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