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Application of the Fuzzy Time Series Chen and Markov Chain Method with Sturges and Average-Based Intervals in Forecasting the Rupiah Exchange Rate Against the US Dollar

Nurkholis[1], Puspita Kartikasari[2], Fariz Budi Arafat[3]

[1,2,3]Department of Statistics, Faculty of Science and Mathematics, Diponegoro University, Semarang, Indonesia.

Corresponding Author: Fariz Budi Arafat

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ABSTRACT

International trade plays a crucial role in meeting the needs of a country's population, with exchange rates being a key factor influencing cross-border transactions. The instability of exchange rate fluctuations necessitates accurate forecasting to assist governments, exporters, and importers in decision making. As the United States is one of Indonesia's largest trading partners, predicting the Rupiah to US dollar exchange rate becomes highly essential. Fuzzy Time Series (FTS) is a linguistic-based forecasting method that is effective for data with no discernible patterns. This study compares the accuracy of two FTS developments, namely FTS Chen and FTS Markov Chain, using two interval determination methods: Sturges and Average Based methods. The results show that the FTS Markov chain with average-based intervals provides a better forecasting performance than the other three methods, achieving a MAPE value of 0.3351%.

Keywords: *Exchange rate, Fuzzy Time Series, Sturges, Average Based, Interval, MAPE.*

INTRODUCTION

International trade refers to export and import activities that involve goods and services. It serves as one of the means for a country to fulfill the needs of its population. Export and import activities across national borders require equivalent exchange values. The exchange rate is used to describe the value of a country's currency relative to that of another country. One of Indonesia's main export destinations is the United States. According to data from Statistics Indonesia (BPS) [1] in 2022 and 2023, Indonesia's exports to the United States ranked second after China.

Forecasting the Rupiah to US dollar exchange rate offers significant benefits for governments, investors, exporters, and importers in making strategic decisions. Forecasting is a technique used to estimate future values based on present or historical data. Forecasting methods continue to evolve across various fields in line with technological advancements. Song and Chissom (1993) [2] introduced a time-series forecasting technique known as Fuzzy Time Series (FTS). FTS is a forecasting technique based on machine learning, in which actual data are transformed into linguistic values using relatively simple algorithms [3].

FTS has continued to develop, with one of its variations being the Fuzzy Time Series Chen (FTS Chen), proposed by Chen (1996) [4]. This method applies a simpler arithmetic-based algorithm than the original FTS. Tsaur (2012) [5] further developed the approach by integrating FTS with the Markov Chain, commonly known as the Fuzzy Time Series Markov Chain (FTSMC). The determination of intervals in the FTS method significantly influences the forecast accuracy [6]. Therefore, selecting an appropriate interval is crucial for improving forecasting performance. Xihao and Yimin (2008) [7] applied an average-based interval approach to forecast the Shanghai Composite Index, resulting in a Mean Square Error (MSE) of 292.3224. Mauludin and Utama (2023) [8] forecasted truck demand in Batam using the Fuzzy Time Series Chen method with the Sturges interval determination, achieving a highly accurate forecast with a Mean Absolute Percentage Error (MAPE) of 4.4%. Based on the above, this study aims to forecast the Rupiah to US Dollar exchange rate using the Fuzzy Time Series Chen and Markov Chain method with Sturges and average-based intervals to compare forecasting accuracy based on the MAPE metric.

LITERATURE REVIEW

The concept of Fuzzy Time Series was first introduced by Song and Chissom in 1993[1]. Over time, the method has undergone various developments; however, several critical issues remain, one of which is the determination of the interval length. The interval length plays a significant role in influencing the forecasting accuracy of the fuzzy time-series method [6]. Several approaches have been proposed for determining the interval length in Fuzzy Time Series, including the Sturges and average-based methods.

1. Sturges

Herbert Arthur Sturges, a mathematician and statistician, introduced a method to determine the number of intervals. The Sturges approach to determining the number of intervals uses Equation 1:

$$k = 1 + 3{,}322 \log(n) \tag{1}$$

where k is the number of interval classes and n is the total number of data points.

2. Average-Based

The determination of intervals using an average-based method was introduced by Huarng (2001) [6]. The procedure for determining the interval length using the average-based method is as follows.

a. Calculate all the absolute differences between A_{i+1} and A_i $(i = 1, \dots, n-1)$ as the first difference and the average of the first differences.
b. Take one-half of the average (in step a) as the length.
c. According to the length (in step b), determine the base for the length of intervals, as shown in Table 1.
d. The length was determined according to the base determined as the length of the intervals.

Table 1. Base Mapping Table

Range	Base
0,1 – 1,0	0,1
1,0 – 10	1
11 – 100	10
101 - 1000	100

Fuzzy Time Series Chen (FTS Chen) is a development of the FTS method, first introduced by Chen (1996) [3]. The following steps must be performed to perform forecasting using the FTS Chen method [3].

1. Define and partition the universe of discourse

The universe of discourse is defined with the lower bound as the minimum value of the historical data minus D_1 and the upper bound as the maximum value plus D_2, D_1 and D_2 are positive constants determined by the researcher. The universe of discourse is partitioned using Sturges and Average-Based methods.

2. Define fuzzy set on the universe of discourse

The fuzzy sets were constructed using fuzzy membership A_i, as defined in Equation 2.

$$A_i = \frac{\mu_{A_i}(u_1)}{u_1} + \frac{\mu_{A_i}(u_1)}{u_1} + \cdots + \frac{\mu_{A_i}(u_n)}{u_n} \quad (2)$$

The membership degree $\mu_{A_i}(\mu_k)$ was determined using Equation 3.

$$\mu_{A_i}(\mu_k) = \begin{cases} 1;\ i = k \\ 0{,}5; i = k-1 \ or \ i = k+1 \\ 0; others \end{cases} \quad (3)$$

3. Fuzzification and Fuzzy Logical Relationship (FLR)
   Fuzzification is the process of identifying and assigning data to fuzzy sets, that is, transforming numerical data into a linguistic form. FLR refers to the relationship or connection between each data point and subsequent data point in the form of fuzzy sets, denoted as $A_i \to A_j$.
4. The derived Fuzzy Logical Relationship is divided into groups (Fuzzy Logical Relationship Group) based on the current states of the enrolment of fuzzy logical relationships.
5. Calculate the forecasted output using the following principles:

a. If the FLRG of A_i has no relation ($A_i \to \phi$), then the forecast value is m_i, where m_i is the midpoint of the interval in which the fuzzy set A_i is located.

b. If the FLRG of A_i has a single relation ($A_i \to A_j$), then the forecast value is m_j, where m_j is the midpoint of the interval in which the fuzzy set A_j is located.

c. If the FLRG of A_i has multiple relations ($A_i \to A_1, A_2, A_3 \dots A_k$),), then the forecast value is calculated using Equation 4.

$$F(t) = \frac{m_1 + m_2 + m_3 + \cdots + m_k}{n} \quad (4)$$

Where $F(t)$represents the forecast value at period t, m_k is the midpoint of the k interval class, and n denotes the number of classes contained in the FLRG.

The fuzzy time-series Markov chain (FTSMC) is a development of the fuzzy time-series method combined with the Markov transition matrix. The FTSMC was first introduced by Tsaur (2012). The steps for forecasting using the FTSMC are as follows (Tsaur, 2012):

1. The process of forming the universe of discourse up to the formation of the FLR is the same as in the FTS Chen process, covering Steps 1 to 3.
2. Determine the FLRG where all identical relations in the FLR must be recorded and the frequency of the same next state must be counted.
3. Determine markov state transition matrices

Markov state transition matrices: n states are defined for each time step for the n fuzzy sets, thus the dimension transition matrix is nxn.

$$\boldsymbol{P} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ P_{31} & P_{32} & P_{33} & \cdots & P_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nn} \end{bmatrix}$$

$$P_{ij} = \frac{H_{ij}}{H_i}; i, j = 1,2,3, \dots, k \quad (5)$$

where P_{ij} represents the transition probability from state A_i to state A_j. H_{ij} denotes the number of transitions from state A_i to state A_j, while H_i is the total number of elements included in state A_i.
If $P_{ij} \geq 0$, then state A_j is accessible from state A_i,
If state A_i and A_j are accessible to each other, then A_i communicates with A_j.

4. Calculated the forecasted outputs
   The expected forecast values were obtained using the following principles:

a. If the FLRG of A_i has a single relation ($A_i \to A_k$) where $P_{ij} = 0$ and $P_{ik} = 1$, with $j \neq k$, then the forecast value is m_k, where k is the midpoint of the interval corresponding to the fuzzy set A_k.
b. If the FLRG of A_j has multiple relationships $A_j \to A_1, A_2, A_3 \dots A_n$, when collected data $Y(t-1)$ at time $t-1$ is in the state A_j, then the forecasting $F(t)$ is equal as $F(t) = m_1 P_{j1} + m_2 P_{j2} + m_3 P_{j3} + \cdots + m_{j-1} P_{j(j-1)} + Y(t-1) P_{jj} + m_{j+1} P_{j(j+1)} + \cdots + m_n P_{jn}$

where $m_1, m_2, m_3, \dots, m_{j-1}, m_{j+1}, \dots, m_n$ are midpoint of $u_1, u_2, u_3, \dots, u_{j-1}, u_{j+1}, \dots, u_n$, and m_j is substituted for $Y(t-1)$.

5. The values were adjusted according to the following principles:

a. If state A_i communicates with A_j, starting in state A_i at time $t-1$ as $F(t-1) = A_i$, and makes an increasing transition into state A_j at time t ($i < j$) then the adjusting trend value $D_{t1} = (\frac{\ell}{2})$.
b. If state A_i communicates with A_j, starting in state A_i at time $t-1$ as $F(t-1) = A_i$, and makes an decreasing transition into state A_j at time t ($i > j$) then the adjusting trend value $D_{t1} = -(\frac{\ell}{2})$
c. If the current state is in state A_i at time $t-1$ as $F(t-1) = A_i$, then makes a jump-forward transition into state A_{i+s} at time t ($1 \leq s \leq n-i$) then the adjusting trend value $D_{t2} = (\frac{\ell}{2})s$.
d. If the current state is in state A_i at time $t-1$ as $F(t-1) = A_i$, then makes a jump-backward transition into state A_{i-v} at time t ($1 \leq v \leq i$) then the adjusting trend value $D_{t2} = -(\frac{\ell}{2})v$.

6. General form for forecasting result $F^*(t)$ can be obtained

$$F^*(t) = F(t) \pm D_{t1} \pm D_{t2} \tag{6}$$

The forecasting accuracy can be measured using the Mean Absolute Percentage Error (MAPE).

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \frac{|Y(t) - F^*(t)|}{Y(t)} \times 100\% \tag{7}$$

where *n* represents the number of data points, *Y(t)* is the actual data at time *t*, and $F^*(t)$ denotes the final forecast value at time *t*. According to Chang et al. (2007) [9], a forecasting result is considered very good if the resulting MAPE (Mean Absolute Percentage Error) is less than 10%

MATERIALS & METHODS

This study utilizes data on the Rupiah to US Dollar exchange rate, which is publicly accessible on the Bank Indonesia website [10]. A total of 400 data points were used, covering the period from January 2, 2023, to September 9, 2024. The dataset was divided into 393 training and 7 testing datasets. The data analysis process carried out in this study consisted of the following steps:

1. Describing the data
2. Developing forecasting models using Fuzzy Time Series Chen with Sturges and Average-Based intervals
3. Developing forecasting models using Fuzzy Time Series Markov Chain with Sturges and Average-Based intervals
4. Calculating the forecasting accuracy of all four methods using the Mean Absolute Percentage Error (MAPE)
5. Comparing the forecasting accuracy values and selecting the best model based on the MAPE score

RESULT

The pattern of the Rupiah (IDR) to US Dollar (USD) exchange rate from January 2, 2023, to September 9, 2024, exhibits a fluctuating trend, characterized by irregular increases and decreases. The minimum exchange rate occurred on May 4, 2023, with a value of 14632, whereas the maximum exchange rate was recorded on June 21, 2024, at 16458. The pattern of the Rupiah to US dollar exchange rate is shown in Figure 1.

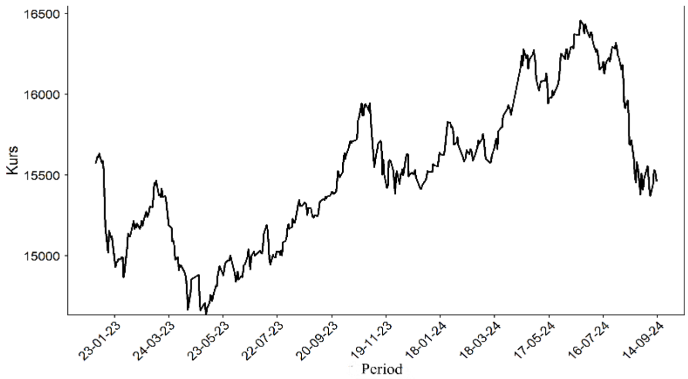

Figure 1. Pattern of Rupiah Exchange Rate Data Against the US Dollar

The following describes the forecasting procedure using the fuzzy time-series Chen method with Sturges and Average-Based intervals.

Step 1. Defining the Universe of Discourse

The minimum value in the historical data was 14632 and the maximum value was 16458, with D_1 and D_2 both equal to 7. Therefore, the universe of discourse was defined as U = [14625; 16465].

Step 2. Defining the Intervals

The determination of intervals using the Sturges method resulted in 10 intervals with an interval length of 184, whereas the average-based interval determination method produced 46 intervals with an interval length of 40.

Step 3. Defining the Fuzzy Sets

Fuzzy Sets Sturges	Fuzzy Sets Average-Based
$A_1 = \frac{1}{u_1} + \frac{0,5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{10}}$	$A_1 = \frac{1}{u_1} + \frac{0,5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{46}}$
$A_2 = \frac{1}{u_1} + \frac{0,5}{u_2} + \frac{1}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{10}}$	$A_2 = \frac{1}{u_1} + \frac{0,5}{u_2} + \frac{1}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{46}}$
$\vdots$	$\vdots$
$A_{10} = \frac{0}{u_1} + \frac{0}{u_2} + \cdots + \frac{0}{u_8} + \frac{0,5}{u_9} + \frac{1}{u_{10}}$	$A_{46} = \frac{0}{u_1} + \cdots + \frac{0}{u_{18}} + \cdots + \frac{0,5}{u_{45}} + \frac{1}{u_{46}}$

Step 4. Fuzzification

Table 2. Fuzzification Sturges and Average-Based Method

t	Data	Sturges Method Fuzzification	Average-Based Method Fuzzification
1	15572	A_6	A_{24}
2	15590	A_6	A_{25}
3	15615	A_6	A_{25}
4	15610	A_6	A_{25}
5	15635	A_6	A_{26}
⋮	⋮	⋮	⋮
393	15409	A_5	A_{20}

Step 5. Defining the Fuzzy Logical Relationship (FLR)

The Fuzzy Logical Relationship (FLR) for the second data point using the Sturges interval determination method is $A_6 \rightarrow A_6$, and for the third data point, the FLR is also $A_6 \rightarrow A_6$..

Step 6. Defining the Fuzzy Logical Relationship Group (FLRG)

Table 3. FLRG Sturges and Average-Based Method

FLRG Metode *Sturges*	FLRG Metode *Average Based*
$A_1 \rightarrow A_1, A_2$	$A_1 \rightarrow A_2$
$A_2 \rightarrow A_1, A_2, A_3$	$A_2 \rightarrow A_3, A_4$
$A_3 \rightarrow A_2, A_3, A_4$	$A_3 \rightarrow A_1, A_4$
$A_4 \rightarrow A_3, A_4, A_5$	$A_4 \rightarrow A_1, A_3, A_4, A_5, A_6$
$A_5 \rightarrow A_4, A_5, A_6$	$A_5 \rightarrow A_2, A_5, A_7$
$A_6 \rightarrow A_5, A_6, A_7$	$A_6 \rightarrow A_7$
$A_7 \rightarrow A_6, A_7, A_8, A_9$	$A_7 \rightarrow A_4, A_5, A_6, A_7, A_8, A_{11}$
$A_8 \rightarrow A_7, A_8, A_9$	$A_8 \rightarrow A_7, A_8, A_9, A_{10}$
$A_9 \rightarrow A_8, A_9, A_{10}$	⋮
$A_{10} \rightarrow A_9, A_{10}$	$A_{46} \rightarrow A_{44}, A_{45}, A_{46}$

Step 7. Defuzzification and Forecasting

The forecasted value or defuzzification for A_1 with the Fuzzy Logical Relationship Group (FLRG) $A_1 \rightarrow A_1, A_2$ using the Sturges interval determination method is

$$F(t) = \frac{m_1 + m_2}{2} = \frac{14717 + 14901}{2} = 14809$$

The forecasted value or defuzzification for A_3 with the FLRG $A_3 \rightarrow A_1, A_4$ using the Average-Based interval determination method is

$$F(t) = \frac{m_1 + m_4}{2} = \frac{14645 + 14765}{2} = 14705$$

The final forecast was obtained by converting the linguistic values of each actual data point back into numerical values based on the defuzzification results. A comparison graph of the training data forecast results using the FTS Chen method with Sturges and Average Based intervals is shown in Figure 2.

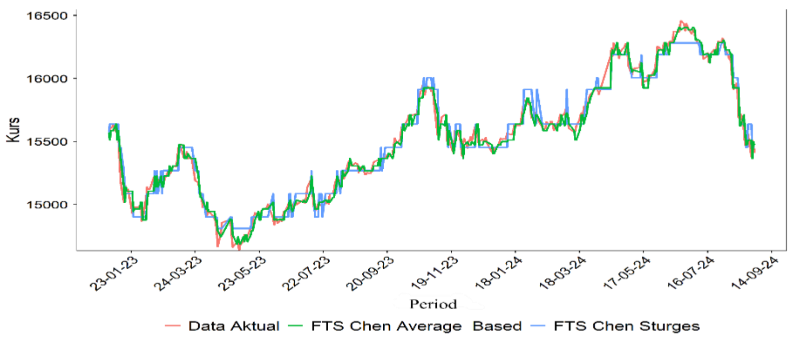

Figure 2. Graph of Training Data and Forecasting Using FTS Chen with Sturges and Average-Based Methods

The data patterns formed between the training data and the forecasts using the FTS Chen method with Sturges and Average-based intervals closely overlap, although in the FTS Chen Sturges method, there are several points that deviate significantly from the training data. However, the majority still followed a similar pattern, indicating that the forecasted values were close to the actual data. The forecasting results using the FTS Chen Sturges and Average-based methods for the next seven periods are compared with the testing data in Table 4.

Table 4. Forecasting FTS Chen Sturges and Average-Based

t	Data	Forecasting FTS Chen Sturges	Forecasting FTS Chen Average Based
394	15.473	15.453	15.405
395	15.536	15.453	15.405
396	15.557	15.453	15.405
397	15.490	15.453	15.405
398	15.410	15.453	15.405
399	15.372	15.453	15.405
400	15.446	15.453	15.405

The MAPE values of the forecasts using the FTS Chen method with Sturges and Average-based intervals were 0.3460% and 0.4744%, respectively. This indicates that both methods produced highly accurate forecasting results.
The modeling steps for the FTS Markov Chain with Sturges and Average-based intervals from Step 1 to Step 3 are identical to those of the FTS Chen method with Sturges and Average-based intervals. The next step is to construct the FLRG in which the frequency of each identical next state must be counted. Using the Sturges interval determination method, the FLRG for A_1 is $A_1 \rightarrow 12(A_1), 2(A_2)$.The following section presents the forecasting results using the FTS Markov chain with sturges and average-based intervals on the training data.

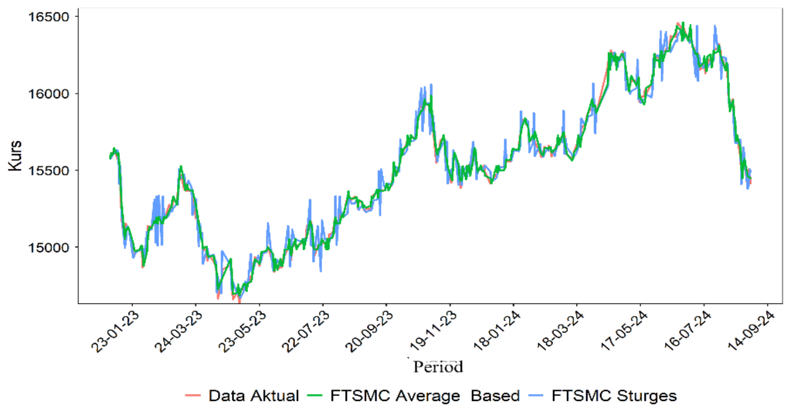

Figure 3. Graph of Training Data and Forecasting Using FTSMC with Sturges and Average-Based Methods

The pattern formed between the training data and the forecasts using the FTS Markov Chain method with Sturges and Average-based intervals overlaps and follows a similar trend. Therefore, it can be concluded that the forecasted values closely approximated the actual data. The following are the forecasting results for the next seven periods using the FTS Markov Chain method with Sturges and Average Based intervals, which will later be compared with the testing data.

Table 5. Forecasting FTMC Sturges

t	Actual Data	Forecast value ($F_{(t)}$)	Adjustment value ($D_{(t)}$)	Final forecast ($F^{*}_{(t)}$)
394	15473	15425.34	0	15425.34
395	15536	15486.97	0	15486.97
396	15557	15439.15	0	15439.15
397	15490	15491.22	0	15491.22
398	15410	15450.82	0	15450.82
399	15372	15494.81	0	15494.81
400	15446	15460.67	0	15460.67

Table 6 Forecasting FTSMC Average-Based

t	Actual Data	Forecast value ($F_{(t)}$)	Adjustment value ($D_{(t)}$)	Final forecast ($F^{*}_{(t)}$)
394	15473	15411.86	20	15431.86
395	15536	15490.11	0	15490.11
396	15557	15488.36	40	15528.36
397	15490	15488.69	40	15528.69
398	15410	15494.34	0	15494.34
399	15372	15494.41	-40	15462.48
400	15446	15489.54	0	15489.54

The MAPE values obtained from the comparison between the testing data and the forecasts using the FTS Markov Chain method with Sturges and Average Based intervals were 0.3639% and 0.3551%, respectively. Therefore, it can be concluded that both the methods produce highly accurate forecasting results.

DISCUSSION

The fuzzy time series Markov chain method with average-based intervals demonstrated superior performance in forecasting the Rupiah to USD exchange rate, achieving a MAPE of 0.3351%. All four tested methods yielded highly accurate results, with MAPE values below 10%. These findings align with previous studies indicating that fuzzy time series can produce highly accurate short-term forecasts for financial time series data.

The results have significant implications for policymakers, investors, and businesses involved in international trade and currency risk management. The FTS Markov chain method with average-based intervals, in particular, could serve as a valuable tool for predicting short-term Rupiah-USD exchange rate movements, thereby aiding in informed decision-making.

These methods exhibit strengths in handling non-linear and uncertain data, which are common characteristics of financial time series. However, limitations should be noted, including the short forecast horizon tested and the potential for decreased accuracy in longer-term forecasts.

Future research directions could include testing these methods on longer forecast horizons, comparing them to other forecasting techniques such as ARIMA or machine learning models, and applying them to other currency pairs or financial time series.

The study underscores the importance of interval determination in fuzzy time series forecasting, as evidenced by the impact of different interval determination methods on forecast accuracy. This observation supports previous findings emphasizing the significance of this aspect in the forecasting process.

CONCLUSION

The forecasting results from all four methods yielded MAPE values below 10%, indicating that the forecasts produced were highly accurate. Among the four, the fuzzy time series Markov chain with an average-based interval is the most effective method for forecasting the exchange rate of the Indonesian Rupiah (IDR) against the United

States Dollar (USD), with an MAPE of 0.3351%.

Declaration by Authors

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