International Journal of Research and Review
Vol. 10; Issue: 1; January 2023
Website: www.ijrrjournal.com
Review Paper
E-ISSN: 2349-9788; P-ISSN: 2454-2237

A Review of Multilevel Modelling and Its Empirics on Frontier Analysis

Peter Chimwanda[1], Philimon Nyamugure[2], Precious Mdlongwa[3]

[1]Chinhoyi University of Technology, Zimbabwe
[2]National University of Science and Technology, Zimbabwe
[3]National University of Science and Technology, Zimbabwe

Corresponding Author: Peter Chimwanda

DOI: https://doi.org/10.52403/ijrr.20230143

ABSTRACT

Multilevel modelling has gained ground in the analysis of clustered data over its counterparts, aggregation and disaggregation. This is evidenced by a recent increase in its application in efficiency analysis, an area that is laden with clustered data. Conventional efficiency analysis models have relied on the reduction of tiered data to a single level by aggregation, disaggregation or, to some extent, ignoring the structure of data by discarding the variables, outside the level of the unit of analysis. The article presents a review of existing literature on multilevel models as an entity. Panel data, a special case of multilevel data, are discussed. Attention is then given to how these (multilevel) models, together with panel data, have found their way in efficiency analysis in literature.

keywords: multilevel, frontier, clustered data.

1.INTRODUCTION

A significant amount of data that is used in efficiency analysis is tied. In local governments, for example, revenue collection is done by wards and records are kept accordingly. Efficiency analyses on revenue collection have been done in this area but in most cases, the multilevel data encountered have been reduced to one level by one method or another. The methods commonly used in this reduction are aggregation and disaggregation.

Aggregation is done by moving variables from the lower level to the higher level, (Hox, 1995), in the case of two-level data. Here, data values from several lower-level units are reduced to fewer higher-level units. Commonly, averages of values of some explanatory variables for every group are calculated. The averages are then used in a standard regression analysis, (Twisk, 2006). This means that micro-level or lower-level variables are moved to the higher level by calculating summary measures to represent them. An analysis is then done at this higher level. In a class of students, for example, the mean can be calculated and is taken to represent the class. The class becomes the unit of analysis and not the student.

Statistical problems created in this kind of analysis include reduction of sample size as well as information and power loss, (Hox, 1995). The non-independence of members within clusters can no longer be studied. The sample size is reduced as several members of a group are reduced to a single group mark, the average. This therefore means less statistical power for such data, (Hox, 1995). Since the group is now the unit of analysis, no inference at the individual level is done. This kind of inference, where analysis is done at one level and conclusions are formulated at a different level may cause a misinterpretation of data, referred to as ecological fallacy. In health studies, for example, it was shown that countries whose staple diet includes fat, have higher rates of women dying from breast cancer, (Carroll, 1975). Taking these results to the individual level, one is tempted to conclude that women who eat a lot of fat have a higher probability of suffering from breast cancer. Suggestions from recent studies, however, have shown that the relationship between fat consumption and breast cancer is not that strong, (Holmes et al., 1999). Nevertheless, aggregation is not a mistake if the researcher is only interested in the macro-level proposition, (Marta, 2010). The reliability of aggregated data however, is governed by the number of higher-level units. Larger higher-level units are more reliable than smaller ones. If the researcher is concerned with both or all the levels, then, aggregation results in errors as it reduces the data to a single level. Other levels are ignored in the aggregation process.

When the lower level unit is of interest, aggregation becomes inappropriate. in local governments, interest is usually on the behaviour of wards and not towns, in revenue collection. Methods that do not ignore the ward are therefore appropriate.

With disaggregation, hierarchical data issues are dealt with by ignoring group differences. Context-free relationships between variables are assumed. Instead of doing the analysis at some higher level, variables are moved to the lowest level by assigning to each lowest level unit, a value that reflects the group to which it belongs, (Hox, 1995). The analysis is, this time, done at the lowest level. This analysis may be multiple regression analysis or analysis of variance or any other standard analysis method, (Hox, 2010). Higher level groups are, therefore, ignored in this kind of analysis which is sometime called naive regression, (Twisk, 2006). Individuals are the units of analysis and not the clusters or groups. Inferences of association at cluster level are made at the lowest level of the structure.

Now that both aggregation and disaggregation have shortfalls, it is advisable to analyse clustered data, using techniques that take cognizance of the structure of the data. When analysing revenue collection in local governments, though the unit of analysis is the ward, the knowledge that these wards fall under municipalities must be made use of and variables at municipality level must be allowed to play a part in model building.

By moving variables to one level, the fact that observations within the same group are correlated is ignored, which results in wrong statistical conclusions (atomistic fallacy). Performing single-level analysis on hierarchical data results in parameter estimates that are inefficient though they are unbiased. Treating clustered data as single level has been proved to lead to the underestimation of standard errors. These standard errors are used in the calculation of test statistics when testing hypotheses and this underestimation leads to an overestimation in the modulus of test statistics, resulting in the inflation of the probability of falsely rejecting the null hypothesis.

In situations where and when the two approaches above are not applied, ignoring the structure of the data becomes the alternative. Those who have attempted to consider multilevel modelling in efficiency analysis, have not taken the models to be frontier as such but have interpreted the errors at each level as a measure of inefficiency at that level.

The next sections of the paper are as follows: Section 2 reviews the literature on multilevel models and is divided into three subsections which are (i) Multilevel models outside efficiency estimation, Multilevel models in efficiency estimation and (iii) Panel data in efficiency estimation and section 3 concludes.

2. Multilevel Models

2.1 Multilevel Models outside efficiency measurement.

(West et al, 2006) define clustered data as data sets in which the units of analysis are grouped or nested within clusters and the response variable is measured once for each subject (the unit of analysis). Modelling such data requires methods other than ordinary analysis of variance or ordinary linear regression analysis. The mentioned approaches assume independence of observations, which does not hold for clustered data, (Galbraith et al, 2010). Multilevel modelling, which is also known by several other names that include Hierarchical modelling and Mixed effects modelling, because of its ability to account for the non-independence of observations in clusters, is considered the most suitable approach for analysing clustered data.

(Gelman and Hill, 2007) define a multilevel model as a regression model in which the regression coefficients are not fixed but are defined by a probability model. Before giving this compact definition, the two first view these models as models that can be expressed in three or more ways that include varying-coefficients models, models with two or more variance components and models with many predictor variables that include indicator variables. In accordance with them, varying coefficients as well as the definition of such coefficients in the form of a model, are the key features of multilevel models. Multilevel models are also called hierarchical models because of the structure of the data that they model, whose units of analysis, which are at the lowest level, are nested in high level units often called clusters, (Gelman and Hill, 2007). The term hierarchical is also derived from the models themselves which form a hierarchy in which the topmost equation has the dependent variable of interest and below it, are equations in which the parameters of the first are subjects which are themselves dependent variables.

Sharing the same view with (Gelman and Hill, 2007) is (El-Horbaty and Hanafy, 2018) who see multilevel linear regression models as a generalization of linear models in which the regression coefficients are themselves modelled from the data. One of the common features of these definitions is that the parameters of these models are not fixed but vary by groups. These models are an extension of regression, (Paterson and Goldstein, 1991) in which data are structured in groups and the coefficients of the models are allowed to vary by groups, (Twisk, 2006).

A unique characteristic of multilevel models is that there is at least one equation at each of the levels of the structured data. At level 1, which is the lowest level and the one where measurement of the dependent variable is taken, there is one equation, with coefficients that are defined by a probability distribution. At level 2, it is the level 1 coefficients which are subjects and are dependent on data at that level. If the coefficients of variables at the second level also vary, then we have third level equations to explain these coefficients and so on.

Among the researchers who studied multilevel modelling, are those who focused much on the type of data that the models are meant for. (West et al, 2006) view clustered data as data sets in which the units of analysis are grouped or nested within clusters and the response variable is measured once for each subject (the unit of analysis). In a discussion about methods of analysing clustered data, (Galbraith et al, 2010) note that the key feature of clustered data is that there is homogeneity within and heterogeneity between clusters, that is, more similarity is reflected by observations within the same cluster than those from different clusters. Examples of clustered data from (Galbraith et al, 2010) are; (i) clinical trials from multi-centres, where the multi-centres are the clusters and patients are the subjects, (ii) observations on children nested within classrooms, (iii) repeated measures, where two or more measurements of the same variable are taken from the same subject(s) over different conditions, with the subjects being the clusters in such experiments, and (iv) longitudinal data, which are a special type of repeated measures data in which the repeated measurements of the same variable(s) are taken over time on the same individual(s). This clustering of data results in the violation of one of the assumptions of standard linear regression modelling, the independence assumption, (Galbraith et al, 2010).

The correlation of observations within the same cluster should be accounted for and it is multilevel models that have the capacity to do that. These models assume different names in different fields. Examples of these names and their respective areas of use are, Hierarchical models, Statistics, (Harville, 1977), random coefficients models, Econometrics, (Swamy, 1970), variance or covariance components models, Experimental design, (Dempster et al, 1981), random-effects models, (Laird and Ware, 1982) or mixed effects models, Biostatistics (Goldstein, 1986). The name random coefficients regression was used by (Swamy, 1970). Rather than deriving the name of the models from variance-covariance effects used by most of the researchers in the area, this author chooses to name the models from a characteristic of their coefficients. The author centres his interest on the variance-covariance component of the models to the extent of presenting a method for the estimation of the variance-covariance matrix, and not the coefficients. (Twisk, 2006), however, sees the name 'random coefficients' as inappropriate for these models and correctly notes that there is a difference between being random and varying by groups and contends that the coefficients of multilevel models are not random as such but vary by groups. This view is shared by (Bickel, 2007) who shuns the terms random coefficients and describes the coefficients as simply varying by groups. In Econometrics, the term 'Random Coefficients Models', is often used to refer to multilevel models, (Swamy, 1970). (Bickel, 2007) clearly distinguishes between random coefficients models and multilevel models. In addition to having random coefficients, multilevel models have independent variables that come from two or more levels. In the analysis of such data, cross-level interaction effects are created and become other variables in the model. The error term of the model becomes complex as it has components from all the levels in the dataset of interest, (Bickel, 2007). The researcher uses several datasets to put forward his account of multilevel modelling as the best models when the power of ordinary least squares is exhausted. Bickel recognizes that, for clustered data, ordinary least squares regression coefficients have standard errors that are deflated to the extent of giving misleading results in inferential statistics.

There is a number of methods that can be used to account for the deflation of standard errors caused by the use of single-level models when data is clustered data. Most of these methods are simpler to grasp and work with than multilevel analysis, (Bickel, 2007). Unlike most researchers who see these models as meant only for accounting for the discrepancy in standard errors, Bickel sees them (multilevel models) accomplishing tasks beyond the capacity of ordinary least squares. Permitting intercepts and slopes to vary by groups, is an example of achievements by multilevel models, which are beyond the scope of ordinary least squares.

Multilevel models allow the independent variables in a regression model to have effects that vary by groups, to the dependent variable of interest. Identification of contextual sources of variability in model parameters as well as their (the sources) incorporation into the model are some of the benefits of multilevel models. Frequently, however, there are similarities between the regression coefficients of multilevel models and those of single level models. It is these similarities that lead researchers to the assertion that multilevel models are just an extension of ordinary least squares regression.

There is a difference between random coefficient regression and multilevel regression analysis. The latter is considered as an application of the former, (Bickel, 2007). This author considers a random coefficient as a regression coefficient that has two components, the fixed and the random. It is the variation of coefficients by groups that the researcher takes as randomness.

For random coefficient models, there is no nesting nor is there context. These models are transformed to multilevel models by the introduction of contextual variables, which are at a different level. It is these contextual variables that give substantive explanations of cluster-to-cluster variability in intercepts and slopes. Both random coefficient models and multilevel models have fixed and random components. Analysts are mostly interested in fixed components, (Bickel, 2007).

Random component values are for inferential purposes like the construction of intervals of fixed components. The intervals in the context of multilevel models are different from the confidence intervals in multiple regression models. Whereas in multiple regression models, intervals gauge the consequences of random sampling error, the confidence intervals in multilevel analysis are for estimating how much intercepts and slopes vary across groups because of the nesting.

The 2004 USA presidential election dataset was used to illustrate how the models work. The dependent variable in this analysis, denoted Y_{BUSH}, was the percentage of people who voted for Bush. The unit of analysis was the county and the nesting units were the states. The independent variables included the percentage of nuclear families, percentage of ethical minorities, percentage of rural residents and percentage of youth voters. To make this data multilevel, the values of these variables were calculated at each of the two levels, county and state, (Bickel, 2007).

To justify the need for multilevel analysis here, the intra-class-correlation was calculated and found to be 0.352 which is too high to ignore and fit ordinary least squares regression to the data. This correlation suggests that 35.2% of the variation in the voting behaviour of the people was due to group(state) differences. Four models, that included an ordinary least squares regression model with interactions of independent variables that are at the same level and multilevel regression model, were run. The results showed little difference in the values of the coefficients but, like expected, the standard errors of the multilevel model were significantly different from those of the other three models.

2.2 Multilevel Models in Efficiency Estimation

Multilevel models are a parametric regression-based approach to data analysis. They are regression-type models that are meant for analysing data that is clustered. They have the capacity of accounting for the non-independence of observations that are clustered. They are, however, a non-frontier method as they allow observations to fall above or below the line of best fit. They do not have the two-component error term that is in the stochastic frontier model. Instead, they have at least one error at each level of the hierarchy. Researchers have estimated these errors in order to use them in performance measurement.

In a study whose main theme was to compare rankings from multilevel models to those of data envelopment, (Johnes, 2006) used sample data from 54,564 graduates who completed their programs in 1993 in 49 universities of the United Kingdom. The graduates used were from pure sciences, social sciences, applied sciences and arts. The researchers assigned weights to the various categories of degrees.

Three multilevel models, which are models that consider the hierarchical nature of data, were run with the students at level 1 and the universities at level 2. These models were, the null, one that took pre-university entry requirements as the only explanatory variable, and one which took all the explanatory variables of concern into account. The independent variables were gender, marital status, place of residence and age. The dependent variable was the weighted degree classification of each student, that took the values; no degree, pass, third class honours, lower second class honours, upper second class honours and first class honours with corresponding weights 1.90;2.00;2.20;2.30;2.45 and 2.85. Of interest in university efficiency measurement context was u_{0j}, the level 2 residual, which was taken to represent university effect. This represents the amount by which each university performance is above or below the mean. In general, these are referred to as group effects, university effects in this context. These effects are estimated as

$$\hat{u}_{0j} = \frac{n_j \sigma_{u0}^2}{n_j \sigma_{u0}^2 + \sigma_{e0}^2} \frac{\sum i(y_{ij} - \hat{y}_{ij})}{n_j} \quad (1)$$

These effects, which are the amounts by which the universities deviate from the mean, are used in ranking the universities. Here n_j is the number of students in university j, σ_{e0}^2 and σ_{u0}^2 are residual variances at levels 1 and 2 respectively and y_{ij} is the dependent variable, which is the weighted degree classification.

To decompose the individuals' data envelopment analysis (DEA) efficiency scores into two components, one attributed to the university and the other to the individual student, (Johnes, 2006) used a method developed by (Thanassoulis and Portela, 2002). The scores attributed to universities were ranked and compared to universities' effects from the multilevel model. The results revealed no significant correlation-ship between rankings of universities from the DEA efficiencies and the university rankings from the university effects of the multilevel models, suggesting that efficiency rankings depend on the technique used to estimate the efficiency

Inefficiency levels are then calculated for level 1 units, which are in this case, the individual students. These are given by:

$$\hat{u}_i = E[u_i|\bar{\epsilon}_i] = \frac{\sigma\lambda}{1+\lambda^2}\left[\frac{\phi(\alpha_i)}{1-\Phi(\alpha_i)} - \alpha_i\right] \quad (2)$$

Where

$\lambda = \frac{\sigma_u}{\sigma_\epsilon},\ \sigma = \sqrt{\sigma_u^2 + \sigma_\epsilon^2}\ \ and\ \ \alpha_i = T\bar{\epsilon}\lambda/\sigma.\ \phi(\alpha_i)$ Is the standard normal density evaluated at α_i and $\Phi(\alpha_i)$, the standard normal CDF (integral from $-\infty$ to α_i evaluated at α_i. These formulae are derived from those in (Jondrow et al, 1982).

Like (Johnes, 2006), (Marta et al, 2011) saw the need of taking the structure of data in model building. Rather than building two different models for comparison, the researchers, in their quest to establish how Italian hospitals performed, applied a two-step procedure in measuring the efficiency of the hospitals. The first step was the development of a 3-level model with time as level 1, patients as level 2 and hospitals as level 3. The dependent variable of concern in their multilevel model was total mortality which was the sum of in-hospital deaths and deaths within 30 days from discharge. They chose to call this variable hope of life. Since this dependent variable is dichotomous with value 1 for death and 0 otherwise, logistic multilevel regression was used.

126 hospitals in Lombardy region, Italy were observed over a five-year period, from 2003 to 2007. Hospital variables included whether or not the hospital has a university teaching part, whether or not the hospital has an emergency facility, whether or not the hospital was a mono-specialist, the number of wards, number of beds, the number of physicians in the hospital, ownership of the hospital, that is, private or public and whether or not the hospital was a profit-making organization. Variables at the patient level were age, gender, length of stay in hospital and diagnosis-related groups, (DRG) which are prospective payment systems.

Hospital level residuals, the $u'_{0j}s$, which are not observable, but are estimated through equation 1, were taken to be the dependent variable in the stochastic frontier. Since these residuals take both positive and negative signs, a transformation was made in order to ensure that they are all positive or zero. This was done by adding the minimum of these to all of them. Only hospital variables were used as explanatory variables in this step. The results of this research showed that privately owned small state-owned big hospitals ranked high in efficiency levels.

(Aiello and Bonanno, 2015) noted that literature about small banks recognizes the embeddedness of these banks in local markets suggesting that environmental factors influence their performance. They put forward the same view as (Johnes, 2006) and (Marta et al, 2011) that single level models cannot properly handle multilevel data, despite the fact that literature is laden with such models. They used multilevel modelling in the measurement of the effect of provincial level factors in the performance of mutual cooperative banks, (MCBs). The dataset that they used was a panel from 2006 to 2011 for each 414 Italian banks from 66 of the 103 provinces. The hierarchy was therefore a 3-level one with time at level 1, MCBs at level 2 and provinces at level 3. Cost efficiency scores were used as the dependent variable in the study. Results of the study revealed that location is significant in explaining the behaviour of banks in Italy. Like other researchers mentioned, (Aiello and Bonanno, 2015), in their publication, discussed efficiency but their model is not a frontier model but just multilevel Their efficiency values came from the standard stochastic frontier model and are the dependent variable in their multilevel model. (Aiello and Bonanno, 2015), like (Marta et al, 2011), used both the stochastic frontier and the multilevel models jointly but differently. Whereas the former started with efficiency values from a stochastic frontier model ignoring context, the latter started with the residuals from a multilevel model accounting for context. For each of these two researchers, the outcome of the first model, residuals, for (Marta et al, 2011) and efficiency values, for (Aiello and Bonanno, 2015), is the dependent variable in the second model.

A stochastic frontier model in the name of a hierarchical approach to stochastic frontier analysis was proposed by Lordan in Ireland. (Lordan, 2009) discusses healthcare data of 39 health centres that are nested in 5 co-ops. The data is panel with period 365 stretching from 01 May to 01 May 2004. Payroll is the dependent variable and is calculated as the price of labour for each centre per day.

The centres offer general practitioner healthcare services and each centre is managed by one co-op. The co-op decides on funding allocations, the number of working hours and the reaction time taken to provide a service for all the centres under it, (Lordan, 2009). Each co-op operates independent of the other co-ops. The operation of the centres under co-ops makes the structure of the service provision two-tiered. The unit of analysis in this study was the centre.

Some of the variables to explain the variation in the payroll of each centre were the number of home visits by the centre personnel, the number of treatment centre consultation, the number of redeye calls, where red eye is the time between 0000 and 0800 and the quantity of doctor advice for each day, all at centre level. To complement the level 1 variables, were those at the co-op level (level 2), and these include the co-op number and the reaction time to provide the service.

The level I model was built, with the relevant random parameters to be modelled at level 2. The level two models, whose subjects were the level 1 parameters were also built. The substitution of the level 2 models into the level 1 model to produce the final equation was properly done.

The results of the analysis of this model were compared to those from three other models. The three were, the ordinary least squares model that ignored the tier structure of the data and two others that considered the tier structured but had some of the variables dropped. The three models differed in the number and types of variables that they took into account. The model with co-op level variables had the smallest inefficiency variance. This suggests that the model with level 2 variables was able to explain some variation in the dependent variable which is taken as part of inefficiency by other models. Though the four models differed in both efficiency levels and rankings, the greatest difference was between the presumed multilevel model and the one that ignored the tier structure of the data. Efficiency levels for the model that ignored the tier structure was very low and the correlation between these two models was also very low. All this suggests that it is advisable to take into account the structure of the data when analysing tiered data.

The multilevel model is a special case of the random coefficient model. The presence of contextual variables creates more new variables in the form of cross-level interactions. It is this interaction together with variables from higher levels, that make multilevel models different from random coefficients models. The coefficients of higher level variables added to the model, together with those of the interactions created are all constant, not random. The level two variables will appear as independent variables in the model if they are in the intercept of the level 1 model otherwise their effect is in interactions. The model by (Lordan, 2009) does no show these features. It is one of our objectives to run a model with the features mentioned above.

2.3 Panel Data in Frontier Modelling

(Pitt and Lee, 1981) were the first to use panel data in frontier modelling citing a number of advantages that come with such data. These include permitting tracking the production function of a firm, making the estimation of individual firms' efficiency possible and making investigation on whether inefficiency of firms is time variant or time invariant possible. They derived the maximum likelihood function of both *v* (the noise component) and *u* (the inefficiency component) of the error term. These were used in the estimation of efficiency levels of Indonesian weaving firms. Their research was followed by (Schmidt and Sickles, 1884) who pointed out that with panel data, assumptions that include that on the distributions of the two errors could be away with. The introduction of panel data to frontier modelling also saw inefficiency being classified as time-variant or time-invariant. With panel data, models can be categorized into groups of those with time-invariant inefficiency, those with time-variant inefficiency those that separate inefficiency and unobserved individual heterogeneity, and those separating persistent inefficiency from heterogeneity and inefficiency levels from these models compared, (Rashidghalam et al 2016). Each of the four classes has both, merits and demerits. (Rashidghalam et al 2016). (Rashidghalam et al 2016) fitted the above models to data on cotton production in some provinces of Iran in the period 2000 to 2010. Labour, seeds and fertilizer were the inputs while cotton yield was the output. Those models separating inefficiency from heterogeneity looked at individual and not group effects. Results of this research showed that technical inefficiency is not affected by time. As an advantage, these models avoid maximum likelihood estimation of parameters by avoiding the distributional assumption of errors. However, no provincial variables were taken into the model, suggesting that the clustering effect of the provinces was not accounted for.

3. CONCLUSION

The paper discussed multilevel modelling, both inside and outside efficiency estimation. All researchers on these models agree that the models have coefficients that vary by group. This variation of coefficients, however, has made some of the researchers in the area take the coefficients as random while others treat random-coefficients and coefficients that vary by group as separate entities. (Bickel, 2007) correctly noted that multilevel modelling is an application of random coefficients models since, for random coefficients models in general, there is no context, implying that explanatory variables come from only one level.

Noted in this review is the way multilevel models have been used in efficiency estimation. Researchers are not turning these models into frontiers but are simply using the errors at level 2 to rank clusters. There is need for research to consider a multilevel frontier model that produces efficiency scores of level 1 units. For real clustered data, the number of clusters is often smaller than that required for multilevel models so there is reason to use simulated data in building this multilevel frontier model.

It is also worth noting that, although by their nature, panel data are clustered, their use in frontier modelling does not change the conventional frontier to a multilevel one. A hierarchical approach to frontier modelling need to retain the current use of panel data and work with data that have three levels, with the repeated measures level as the lowest level. The unit of analysis here remains the individual, which in this case has another level above it.

Declaration by Authors

Acknowledgement: The researchers appreciate the constructive suggestions from the referees for improvement of this paper.
Source of Funding: None
Conflict of Interest: The authors declare no conflict of interest.

REFERENCES

1. Aiello F. and Bonann G., (2015) Multilevel empirics for small banks in local markets, Online at http://mpra.ub.uni-muenchen.de/64399/ MPRA Paper No. 64399, posted 17. May 2015 04:54 UTC.
2. Bickel R. (2007) Multilevel Analysis for Applied Research, It's just regression! TheGuilford Press, New York.
3. Caroll, K. (1975), Experimental evidence of dietary and hormone-dependent cancers, Cancer Research 35, 3374-3383.
4. Dempster, A. P., Rubin, D. B., and Tsutakawa, R. D. (1981). Estimation in covariance components models. Journal of the American Statistical Association, 76, 341–353.
5. El-Horbaty Yahia S. and Hanafy Eman M. (2018) Some Estimation Methods and Their Assessment in Multilevel Models: A Review, Biostatistics and Biometrics open access journal volume issue 3.
6. Galbraith S., Daniel J. A. and Vissel B. A Study of Clustered Data and Approachesto Its Analysis, J. Neurosci., August 11, 2010 30(32):10601–10608 10603.
7. Gelman A. and Hill J. (2007) Data Analysis Using Regression and Multilevel/Hierarchical Models, Cambridge University Press, New York
8. Goldstein, H. (1986). Multilevel mixed linear analysis using iterative generalized least squares. Biometrika, 73, 43–56.
9. Harville, D.A. (1977) Maximum Likelihood Approaches to Variance Components Estimation and to Related Problems. Journal of the American Statistical Association, 72, 320 338. http://dx.doi.org/10.1080/01621459.1977.10480998
10. Holmes, M. D., Hunter, D. J., Colditz, G. A., Stampfer, M. J., Hankinson, S. E.,Speizer, F. E., Rosner, B., and Willett, W. C. (1999). Association of dietary intake of fat and fatty acids with risk of breast cancer. Journal of the American Medical Association, 281, 914-920.
11. Hox J.J. (1995) Applied Multilevel Analysis, TT-Publikaties, Amsterdam.
12. Johnes J. (2006b) measuring efficiency: a comparison of multilevel modelling anddata envelopment analysis in the context of higher education, Blackwell Publishing Ltd, Bulletin of Economic Research 58:2, 2006, 0307–3378.
13. Jondrow, J., I. Materov, K. Lovell and P. Schmidt, "On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model," Journal of Econometrics, 19, 2/3, 1982, pp. 233-238.
14. Laird, N. M., and Ware, J. H. (1982). Random-effects models for longitudinal data. Biometrics, 38, 963–974.
15. Lordan G.. Considering Endogeneity, Quality of Care and Casemix- A Hierarchical Random Parameters Approach To Measuring Efficiency For Out of Hours Primary Care Services in Ireland. Applied Economics, Taylor Francis (Routledge), 2009, 41 (26), pp.3411-3423.
16. Marta A., Giorgio V. and Gianmaria M. (2011) Multilevel and Stochastic Frontier Models: A comparison and a joint approach of their performances when investigating panel data, Universit'a degli Studi di Milano – Bicocca.
17. Patterson, L., and Goldstein, H. (1991). New statistical methods for analysing social structures: An introduction to multilevel models. British Educational Research Journal, 17, 387–393.
18. Pitt, M., and Lee L. F. (1981) The Measurement and Sources of Technical Inefficiency in the Indonesian Weaving Industry. Journal of Development Economics 9(1/August), 43-64.
19. Rashidghalam M., Heshmati A., Dashti G. and. Pishbahar E (2016) A Comparison of Panel Data Models in Estimating Technical Efficiency, The Institute for the Study of Labor (IZA).
20. Swamy, P. A. V.B. (1970). Efficient inference in a random coefficient regression

model. Econometrica, Vol. 38, No. 2, pp. 311-323.
21. Schmidt P. and Sickles R.G., (1984). Production frontiers and Panel Data. Journal of Business and Economic Statistics 2(4/October), 367–374.
22. Thanassoulis, E., and Portela, M. D. C. A. S. (2002). School Outcomes: Sharing the Responsibility Between Pupil and School. Education Economics, 10(2), 183–207.
23. Twisk J. W. R. (2006) Applied Multilevel Analysis. A Practical Guide, Cambridge University Press, New York, United States of America.
24. West B. T., Welch K. B. and Gal ecki A. T. (2006) LINEAR MIXED MODELS A Practical Guide Using Statistical Software SECOND EDITION, CRC Press, New York

How to cite this article: Peter Chimwanda, Philimon Nyamugure, Precious Mdlongwa. A review of multilevel modelling and its empirics on frontier analysis. *International Journal of Research and Review*. 2023; 10(1): 389-398. DOI: *https://doi.org/10.52403/ijrr.20230143*

******